Un-topical review
Heisenberg’s dog and quantum computing

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Abstract. A Heisenberg’s dog is the central unit in a quantum computer that
factors prime numbers. We report the outcome of a proof-of-principle Gedan-
kenexperiment in which the number 3 was factored and good agreement was
found both with an analytical theoretical prediction and with the results of a
quantum Las Vegas calculation. Other future applications of Heisenberg’s dog
are hinted at. The anonymous reports of the referees and our reply to them are
reproduced in appendix A and appendix B respectively.

1. Introduction
Ever since its birth in 1935, Schrödinger’s [1] cat has received much and well
deserved attention. The original, very complicated set-up that consisted of a
radioactive sample, a Geiger counter, a hammer, a cell of poisonous gas, etc.—in
addition to the cat itself—gradually evolved into arrangements that are consider-
ably simpler and experimentally more accessible. At the present state of cat
evolution it has become reduced to the superposition of two slightly excited
coherent states of opposite phase. The amount of theoretical work behind this
reduction cannot be overestimated!

Let us break a lance for historical justice and recall that this extraordinary
theoretical achievement was triggered by two developments. First, it has become
increasingly difficult to obtain permission for experiments on live objects [5].

1 For recent developments see [2, 3].
2 We suspect, but cannot prove the case, that Lamb’s [4] verdict about Schrödinger’s
cat, namely

It is a messy, rather pointless problem, and in bad taste for several different reasons. (1)
The contemplated treatment of the cat is inhumane. (2) The possible death of the cat
serves no useful scientific purpose. (3) Cats often have nine lives. (4) A different kind of
bad taste involves proposing a complicated problem when a simple one will suffice to
illustrate the physical principles involved.

provided an additional impetus.
Second, at the current level of funding for fundamental physical phenomena, it is impossible to cover the fee requested by the veterinarian, who is needed to carry out the measurement at the final stage of any Schrödinger’s cat experiment.

It is remarkable, however, that, together with the said simplification, the cat, originally a rather neutral object, has developed two sexes (figure 1). This discovery, generally credited to Janszky et al. [6], brought about the possibility of mating these objects, and the offsprings at their initial state are termed Schrödinger’s kittens. It is also very interesting that, when mature Schrödinger’s cats are superimposed, there is a tendency towards constructive interference in same-sex superpositions and toward largely unpredictable, but often destructive interference in opposite-sex superpositions. According to Janszky, the leading expert [6] on cat sexology, the implications of this disturbing observation are far from being fully understood. Domokos and Janszky [7] have observed that the state resulting from an opposite-sex superposition strongly depends on whether the Schrödinger’s cats are parallel or perpendicular. Further studies are called for, but they are beyond the scope of the present contribution.

The ageing of Schrödinger’s cats has also been at the focus of theoretical and experimental studies [8], so that all aspects of an individual cat’s life have been covered. It has been noticed that the sexual differences disappear as Schrödinger’s cats grow old, and senior cats are indistinguishable from the point of view of maleness and femaleness. This phenomenon, the existence of which in other mammals is vehemently denied by those involved in practical studies, therefore is surely a quantum phenomenon. The experts speak of decoherence [9,10]. For more vulgar minds it is the loss of ability to form a superposition state.

In marked contrast with the scientific scrutiny enjoyed by Schrödinger’s cat, Heisenberg’s dog3 (HD) attracted considerably less attention. Its discovery is just

3 Gajzenberga Gund in the Russian literature; see the textbook series by Landau and Lifshitz. Admittedly, we do not know which volume should be consulted, but our Russian colleagues are positive that everything can be found in Landau and Lifshitz.
as well documented\textsuperscript{4}, and so it will suffice to recall briefly the physical properties of HD. It is so fuzzy that, when Professor Heisenberg walked his dog at the natural speed of the Planck stroll, that is $2\text{ ft s}^{-1}$ \textsuperscript{11} or, for senior citizens, $9.48 \text{ in s}^{-1}$\textsuperscript{12}, with an uncertainty of $10^{-7} \text{ ft s}^{-1}$ (the exponent is fuzzy), the dog’s position became so uncertain that it was impossible to tell the dog from its fuzz.\textsuperscript{5} The velocity uncertainty originates, of course, in the random up-and-down and side-to-side motion of the dog’s tail (incidentally, an early manifestation of quantum chaos \textsuperscript{14}), for which the quantum optics community has adopted the expression ‘waving its tail’. The momentum uncertainty is thus the measure of the dog’s waviness whereas the position uncertainty is clearly related to fuzziness.

The recent advance in the art of quantum computing resulted from the observation that Schrödinger’s cats are good at doing arithmetic. Indeed, all quantum computing schemes employ Schrödinger’s cats of various races, sizes and shapes. They promise the realization of one of the oldest dreams of mankind: the fast factoring of large numbers into primes.

The technical problems faced by those building quantum computers are formidable \textsuperscript{15} and the field is highly competitive. There is not only strong competition among various groups sharing the same goal, but also the little-spoken-about race against research on other topics. The least-spoken-about issue concerns the next industrial revolution: what will first be found in every household, a quantum computer or a quantum teleporter?

Inasmuch as HD is complementary to Schrödinger’s cat, it is hardly surprising that HDs will eventually solve another problem of great cultural value and utmost practical importance, namely the factorization of small prime numbers into nonintegers. A first step towards this goal is reported in the present contribution.

Although it is beyond the scope of the current review, we should mention, for completeness, that research is in progress on a complete history of HD with special attention to its cultural conditioning.\textsuperscript{6}

Here is an outline of the paper. In section 2 we report the set-up of the Gedankenexperiment used for our studies of HD. For reasons analogous to those that led to the great simplification of the concept of Schrödinger’s cat mentioned above, we went through much trouble to reduce the HD concept to something more mundane. The utter simplicity that characterizes present-day Schrödinger’s cats has not been achieved as yet, but we are confident that the readers will appreciate the progress that has been made. This is followed, in section 3, by a comparison of the experimental results with theoretical predictions. Good agreement was reached both with an analytical formula and with a quantum Las Vegas calculation. We hint at possible future applications (section 4) and close with a

\textsuperscript{4} It can be traced to some serious after-dinner conversations between perfectly sober physicists, among them the current authors and some others who, owing to an overdose of modesty, prefer to remain anonymous.

\textsuperscript{5} To the best of our knowledge there is no photographic document showing the dog and its fuzz. We proudly report, however, that Heisenberg’s dog was heard barking in Hungary recently \textsuperscript{13}.

\textsuperscript{6} It has recently been pointed out by Sokal \textsuperscript{16} that the findings of scientific studies are much dependent on the cultural context in which the investigations are carried out. A striking example is the value of Planck’s constant; physicists of one cultural background insist that $\hbar = 10^{-27}$ erg s, and others are convinced that $\hbar = 1$. 
2. Experimental set-up

We shall now describe our experimental set-up (figure 2). It is well known that one-level atoms and two-level atoms are not sufficiently fuzzy to allow for the construction of superposition states of the HD kind. Fortunately, three-level atoms of the \( V \) type are perfectly suited but, since we need essentially a two-level system at one stage of the experiment, we must dress the atom so that one of the excited states is turned into a dark state when forming the HD. Thus we need a device that effects the transition

\[
\text{sad head} \rightarrow \text{fuzzy superposition of } \begin{array}{c}
\text{sad head} \\
\text{happy face}
\end{array} \quad \text{(1)}
\]

Note the appearance of the two orthogonal pirate states.\(^7\) Ödmann [17] has shown recently that the desired transition is obtained when the three-level atom is passed through a Fermi–Dirac condenser (FDC), operated in reverse mode.\(^8\) The fuzzy superposition resulting from the action of the FDC does not, however, possess all the essential properties of a HD. Therefore, we pass it through a down-under booster (DUB), which transfers non-local momentum without applying a force [18]. As a consequence of the force-free momentum transfer, the three-level atom has now really become a HD or, more precisely, a HD on a conveniently extended leash (HD on CEL).\(^9\)

The stage is now set for the factorization experiment. More specifically, in this

\(^7\) We have no idea why dressed three-level atoms with one dark excited state are said to be in a ‘pirate state’; it is one of the many mysteries of the jargon of quantum optics.

\(^8\) The original FDC would put two spin-\( \frac{1}{2} \) particles into the same orbital state. It effects, for instance, the formation of a helium atom in its ground state if two electrons and an \( \alpha \) particle happen to be at hand. Ödmann discovered that reversed-mode operation enables the experimenter to put one particle into a fuzzy superposition of two states. In the case of a three-level atom these are obviously the pirate states.

\(^9\) Any similarity to other pre-existing acronyms, such as HD for Humpty Dumpty or CEL for correlated emission laser, are unavoidable and not unintended. The possibility of CEL was recognized for the first time by Scully [19]. A medieval poem on a HD is on record (see [20]).
proof-of-principle *Gedankenexperiment* we attempt to factor the number 3 into two factors. The two-factor situation requires a two-state HD which is prepared at the initial stage of the experiment, as described in the preceding paragraph. Now, since the number 3 is to be factored, the HD must play the quantum *Hütchenspiel* with three *Hütchens*. Extensions to factoring larger numbers into more factors, such as 137 into 44 factors, are straightforward. Here, the three *Hütchens* are realized by three consecutive microwave resonators $R_1$, $R_2$ and $R_3$ with a photon stored in one of them, which one being unknown to the experimenter. The HD runs through the resonators and finds the photon with the aid of a quite normally destructive (QND) measurement. QND measurements have been reviewed recently by Braginsky and Khalili [23].

At the final stage of the experiment, the state of the HD is determined by a protective measurement (PM) [24]. The result of this measurement is determined by one of the factors, the other factor is given by the amount of non-local momentum that the DUB has transferred.

Finally, we recycle the three-level atom for another run of the experiment. For the sake of reproducible initial conditions, the HD must not remember the previous run. We therefore apply a quantum eraser (QE) (introduced in [29]; not shown in figure 2) to the three-level atom. Control *Gedankenexperiments* have shown that complete amnesia can be achieved.

3. Results

Our results are summarized in figure 3. We should perhaps remind the reader that one outcome of the *Gedankenexperiment* (displayed as a cross (+)) corresponds to the generation of the $x$ and the $y$ coordinate. We note that the agreement with theory (solid line) is marvellous. The solid line represents our analytical result, namely
\[ yx = 3; \]
according to a back-of-the-envelope calculation it should be well approximated by a straight line in a log-log plot.

Somewhat less satisfactory is the comparison with the $x$, $y$ pairs («) produced by a quantum Las Vegas calculation. We employed two personal computers, each armed with a 586 processor. For the purpose of parallel computing, their keyboards were meticulously aligned on an optical bench. It appears that the

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10 We recall that in the classical *Hütchenspiel* (English translation, shell game) a little ball is hidden under one of three thimbles. Whereas the classical variant is characterized by the property that the ball is never under the thimble lifted by the player (this statement is supported by experimental data in abundance [21]), the quantum *Hütchenspiel* exhibits the opposite behaviour, that is the photon is always in the resonator in which you look for it [22].

11 The PM is known as a ‘quantum condom’ (QC) in the quantum optics jargon [25]; we disapprove of such frivolous abuse of the language. Some people claim that the QCs have big holes [26–28], but we prefer to ignore these findings.

12 It is a familiar fact that a PM recognizes the dog by its paw (*ex ungue canem*).

13 This is a quantum-mechanical result. Remember to read it from right to left: 3 equals $x$ times $y$.

14 According to Mebody *et al.* [30], a quantum Las Vegas calculation is similar, in spirit, to a quantum Monte Carlo calculation [31, 32]. The latter was not applicable here because we could not incorporate the FDC and the DUB into the existing schemes.
quantum Las Vegas data are suffering from a small systematic error of unknown origin (grossly exaggerated in this log–log plot). We have excluded all known sources of error (tidal forces, anisotropy in the 3 K background radiation, fluctuations in the solar-neutrino flux, misalignment of the keyboards, etc.) and are at present considering the unlikely possibility of a malfunction of the 586 processors.

We have not indicated any statistical error bars in figure 3, simply because they are not available. After the action of the FDC the system is subject to the Pauliverbot so that each outcome can occur once at most, and we refuse to do a statistical analysis under these circumstances.

4. Future applications

In addition to the factorization of small prime numbers, HDs are also very good at detecting loopholes hiding vainly (LHVs) in theories aimed at replacing standard quantum theory. A crucial test is whether the Bellungleichung (German for ‘bark inequality’)\(^{15}\)

\[
D^2 + |C|^2 \leq 1,
\]

is obeyed. This inequality [33] sets bounds on the dogness \(D\) that measures the amount of the Heisenberg picture present in a given quantum-theoretical

\(^{15}\) Traditionally, many nouns referring to fundamental issues are taken over from the German language in which they are defined most easily. Translations into other languages are unavoidably imprecise. Perhaps unnecessarily we remind the reader of the best-known example; a Welcherweggedankenexperiment is an obvious notion, but who could make sense of ‘which way thought experiment’ without professional training?
treatment, and the (complex) catness $C$ that quantifies the relative presence of the Schrödinger picture. An illustrating example is Dirac’s interaction picture, where one would ideally have $D = |C| = 1/2^{1/2}$. Needless to say, HDs can bark and are therefore predestined for testing the validity of the Bellungleichung. We are at present involved in experimental studies of these matters and shall report results in due course.

5. Summary

We have shown that a three-level atom in a HD state can be used for a reliable factorization of small prime numbers. Extensions to large prime numbers are straightforward. As soon as the Schrödinger’s-cat-based quantum computers are equally successful in factoring large numbers into primes, the complete factorization problem will be solved. Further, we have pointed out that HDs—a race traditionally bred in the mist of Copenhagen—are particularly well suited to testing alternatives to standard quantum theory.

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Appendix A: Anonymous referee reports by Axel Schenzle and Michael Fleischhauer

A.1. (Excerpts from) the first referee’s report

Since the end of the last century the evolution of natural sciences has marched on a street of success without precedent. The reason is that science has devoted its undivided attention to every problem (small or large, important or seemingly dull) that has attracted its attention. Erwin Schrödinger with his unappeasable interest for pussy-cats opened a new line of research to the so far undeservedly neglected animal kingdom.

In view of this success story of the twentieth century, it is entirely inexcusable that important other creatures have been so pitilessly neglected. Where is the quantum theory of non-localizable pigs or quantum-jumping sheep? Where is the theoretical progress on man’s favourite companion, the dog? The existence of HD has remained entirely in the dark for half a century. It has always been argued that it is the enormous fuzziness of this peculiar dog that has made its investigation so extremely difficult, but these are nothing but evasions which should not divert from the incompetence and unwillingness of scientists to deal with this problem so far.

The ultimate reduction of Schrödinger’s cat to two particles by Bohm is an epoch-making triumph of science. It cannot be held against the present paper that HD has not yet been simplified to its very essence. Unfortunately Bohm’s holistic disciples, who meanwhile worship the ‘hiddenness’ of physical concepts, are of no
help in simplifying the entangled-dog concept either. The HD is a miraculously aesthetic concept by itself that satisfies all theoretical desires but, in addition, it has the enormous potential for revolutionizing mathematics. Fellow mathematicians, *cave canem!* While Schrödinger’s cat can be trained to do various useless things such as factoring large numbers into primes, the HD will be able to do quite the opposite; it will be able to factor small primes into large numbers.

The authors demonstrate unexpected talents and a long-standing experience in interpreting experimental and numerical results. Their convincing and ingenious interpretation of figure 3 opens our eyes and makes it immediately obvious that the agreement is an overwhelming proof of the presented theory and not only a rule(r) of thumb.

At this point it cannot be swept under the rug that the paper is also plagued by a number of serious shortcomings.

(i) While the manuscript is written mostly in English, it uses an abundance of technical terminology and quantum-optical jargon mostly borrowed from German. This is entirely inexcusable and makes the paper very hard to read, especially to those who understand neither of the two languages. The authors should eventually stick to a single language, most preferably to Finno-Ugric since it cannot be comprehended by a sizeable number of people and therefore gives nobody an unfair advantage.

(ii) The authors have trundled into the mud by advocating the use of contraceptive means—not to use the vulgar terminology of the paper. We thoroughly refuse the use of such means in complete posterior agreement with the *Encyclica Humanae Vitae.*

(iii) The lack of moral standards of the authors also becomes obvious in the vague and suggestive discussion of superposition states. Parental guidance must be ensured before general distribution of the paper.

(iv) The ridiculous discussion on the value of fundamental constants such as $\hbar$ and $c$ does not fit the high scientific quality of the paper. Everybody has known since childhood that $\hbar = c = 1$. However, attempts to prove that also $2\pi/6=1$ have failed badly, in spite of the fact that it had been defined this way by legislative action in the State of Indiana in 1897 [34].

(v) By mentioning the name of the present referee in the acknowledgments of the paper, the authors have attempted to influence the referee and his independent opinion in an undue way. Here they meet with a man of immutable and uncorruptible standards who is not to be affected by such a blunt attempt. In any case, however, I include (on a separate sheet¹⁶) my bank account number and suggestions for a generous contribution to the funding of my otherwise badly supported research. Depending on the generosity of the transactions, I will suggest immediate publication or rejection of the paper.

I hope that I have been of help to the Editor in deciding this matter. If there are any further questions, call on somebody else. In case the Editor decides not to follow my well substantiated advice, he is entirely responsible for the consequences.

¹⁶ Not reproduced here.
A.2. (Excerpts from) the second referee’s report

I very much enjoyed reading this manuscript. It is sound and I believe that it is a very important contribution to theoretical quantum optics and will open up new areas of research. I strongly recommend the publication of this interesting and very original work.

The authors present for the first time a proof-of-principle Gedankenexperiment to factor the prime number 3 and in this way boldly go where no one has gone before. To this end they propose and study a novel quantum computer based on HD states. The rule-over-the-thumb result together with the analytical calculations showing the different factorizations of 3 in figure 3 are convincing. Only the numerical Montevideo simulations are somewhat dissatisfying. Nevertheless, I believe that also the Monteverdi simulations are in support of the authors’ results. I am convinced that the HD states will soon gain the same growing attention which is now paid to the famous cat states of Schrödinger, which are complementary to the first.

Finally I would like to remark on a seemingly side aspect of the paper, the pirate states. Astonishingly these important non-classical states of the radiation field have not been discussed in the quantum optics literature so far. Not only is it because according to Ödmann [17] they are produced in the reverse-mode operation of a FDC and because they are fundamental blocks for the HD states that makes them appealing, but also they are interesting in their own right. I am convinced that the discussion of the pirate states in the present manuscript will trigger a large number of further publications studying the properties of these states. For example, I believe that an analysis of the $Q$-, the Wigner- and other $s$-parametrized phase-space distributions will yield interesting and potentially useful insights into these fascinating states and will help us to understand their subtle nature. It may lead to a whole new field, to ‘quantum piracy’. But of course, these are all things which go much beyond the scope of the present paper and should be understood as suggestions for future work.

All in all, despite the small weaknesses in the Montessori simulations, this is a very nice paper, which should definitely be published.

Appendix B: Authors’ reply to the referees

We are grateful for the referees’ efforts and are pleased by their positive recommendations. To their minor objections we respond as follows.

To the first referee,

(i) we have naturally adopted the lingua franca of modern science, namely broken English as canonized by Casimir [35].

(ii) and (iv) We are bewildered. How could a person of such high moral standards, as witnessed by his support of the Encyclica Humanae Vitae, argue against the usage of the biblical value of $\pi$?\footnote{And he made a molten sea, ten qubits from one brim to the other; it was round all about, and his height was five qubits; and a line of thirty qubits did compass it round about.’ (I Kings, 7, 23 [36]) implies that $\pi = 3$. We have replaced the archaic spelling ‘cubit’ by the more appropriate modern spelling.}

(iii) Honi soit qui mal y pense.
(v) We value the anonymity of the referees very highly. In other words, we cannot afford to support everybody mentioned in the acknowledgments.

To the second referee, Montevideo? Monteverdi? Montessori?—is the referee implying something?

We think that these brief remarks are properly responsive to the referees’ mild objections and see no need for modifications of or additions to the text as submitted originally. We would like, however, to express our disappointment about the absence of any comment by the referees on our novel method of performing parallel computing. Therefore, we shall, for the time being, not reveal how we can do orthogonal computing.

References
[36] The Bible.